

RAMAKRISHNA MISSION VIDYAMANDIRA

(A Residential Autonomous College)

Belur Math, Howrah

B.A./B.Sc. 1st Semester (July – December 2010)

Mid-Semester Examination, September 2010

Date: 10.09.2010

Mathematics (General)

Full Marks 25

Time: 11 am – 12 noon

Group - A

1. Answer any **one**: 2
 - a) Correct or justify: $\text{Arg}(Z_1 Z_2) = \text{Arg} Z_1 + \text{Arg} Z_2$, Z_1 and Z_2 are complex numbers.
 - b) Verify true or false: i^i is a complex number.
2. Answer any **one**: 1
 - a) Find the argument of $Z = 0$.
 - b) Find $\text{Log } i^2$.
3. Answer any **two**: 3x2
 - a) Express $\text{Log Sin } (x + iy)$ in the form of $A + iB$.
 - b) If $x_r = \cos \theta_r + i \sin \theta_r$, $r=1,2,3$ and $x_1 + x_2 + x_3 = x_1 x_2 x_3$, show that $\sum \cos(\theta_1 - \theta_2) = -1$.
 - c) If $\cos \theta = \frac{1}{2}(a + \frac{1}{a})$ and $\cos \phi = \frac{1}{2}(b + \frac{1}{b})$, then show that $\cos(\theta + \phi)$ is one of the values of $\frac{1}{2}(ab + \frac{1}{ab})$.

Group - B

- Answer any **two** questions: 2x4
4. a) If for three sets A, B, C , $A \cup B = A \cup C$ and $A \cap B = A \cap C$ then prove that $B = C$. 3
b) Define composition of two mappings. 1
 5. Let H be a non-empty subset of a group G . Show that H is a subgroup of G iff $a, b \in H \Rightarrow ab^{-1} \in H$. 4
 6. a) Prove that a group G is abelian if $x = x^{-1} \forall x \in G$. 2
b) In a ring $(R, +, \cdot)$, prove that $a \cdot 0 = 0 \forall a \in R$ where '0' is the additive identity in R . 2

Group - C

7. Answer any **two** questions: 2x4
 - a) Show that $\lim_{x \rightarrow 0} \cos \frac{1}{x}$ does not exist.
 - b) If $f(x) = 1 + x$, $0 < x < 1$
 $= 2 - x$, $1 \leq x \leq 2$
 $= x - \frac{x^2}{2}$, $x > 2$,
examine whether f is differentiable at $x = 1$. Is the function continuous at $x = 1$? Justify.
 - c) If $y = \frac{1}{x^2 + a^2}$, then show that $y_n = \frac{(-1)^n n!}{a^{n+2}} \sin^{n+1} \theta \sin(n+1)\theta$, where $\theta = \tan^{-1} \frac{a}{x}$.